HYPERGEOMETRIC FUNCTIONS
AND THEIR ARITHMETIC AVATARS

Generalized hypergeometric series fully justify their belonging to special functions through appearance in different parts of mathematics and physics. Together with their generalizations, including recently introduced hypergeometric motives, they play an important role in arithmetic geometry. In this course, we will explore this multi-functionality of hypergeometric objects from various perspectives including related computable hypergeometric Galois representations and corresponding L-functions. In particular, we will focus on recent developments that connect hypergeometric functions with motives to count on algebraic varieties and with automorphic forms.

Topics:

(a) Classical (generalized) hypergeometric functions, their summation and transformation formulas (including Clausen's formula). Monodromy of the hypergeometric differential equation, its connection with modular forms and with counts on algebraic varieties, Legendre relations. Formulas for $1/\pi$, CM evaluations.

(b) Finite hypergeometric functions - also called hypergeometric functions over finite fields, their summation and transformation formulas. A concept of hypergeometric motive, its L-function.

(c) p-Adic aspects of hypergeometric functions: Ramanujan-type (super)congruences, Dwork's unit root, residue sum identities for error term analysis.

(d) Hypergeometric functions and modular forms: explicit hypergeometric modularity methods.

(e) q-Deformation of hypergeometric functions, applications to congruences and point counting.

Complex analysis (including the gamma function and Eulerian integrals), basics of Galois theory, Gauss (character) sums are considered as prerequisites; some knowledge of modular forms and elliptic curves will be extremely helpful.