Exploring via the Explicit Hypergeometric Modularity Method (EHMM)

1. Welcome to EHMM! Please visit [1] and try the EHMM calculator.

2. Let $\Gamma = [4, -1, -1, -1, -1]$. Write down the corresponding Laurent polynomial, compute its GKZ differential equation, using [3] by Batyrev.

3. Using $t_4 = \dfrac{256\eta(\tau)^{24}\eta(2\tau)^{24}}{(\eta(\tau)^{24} + 64\eta(2\tau)^{24})^2}$, compute the q-expansion for

$$ {}_3F_2 \left[\begin{matrix} \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \\ & 1 & 1 \end{matrix} ; \, t_4 \right], \quad \text{and} \quad {}_3F_2 \left[\begin{matrix} \frac{1}{4} & \frac{1}{2} & \frac{3}{4} \\ & 1 & 1 \end{matrix} ; \, t_4 \right] \frac{q dt_4}{dq}, $$

   check what kinds of modular forms do you get. Are they cuspforms?

4. Use the EHMM method [2] to write down the integrals corresponding to

$$ {}_4F_3 \left[\begin{matrix} \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & \frac{1}{2} \\ & 1 & 1 & 1 \end{matrix} ; \, 1 \right] \quad \text{and} \quad {}_4F_3 \left[\begin{matrix} \frac{1}{4} & \frac{1}{2} & \frac{3}{4} & \frac{1}{4} \\ & 1 & 1 & \frac{3}{4} \end{matrix} ; \, 1 \right]. $$

   What do you get if you use commutative formal group law? How do you compare unit roots for both cases? [Note that the second case is imprimitive and is not defined over $\mathbb{Q}$.]

5. If the datum is changed to

$$ {}_4F_3 \left[\begin{matrix} \frac{1}{4} & \frac{3}{4} & \frac{1}{3} & \frac{2}{3} \\ & 1 & 1 & 1 \end{matrix} ; \, 1 \right], $$

   any idea how to get to the modular form explicitly?

6. What are p-adic the pattern for ${}_4F_3 \left[\begin{matrix} \frac{1}{4} & \frac{3}{4} & \frac{1}{3} & \frac{2}{3} \\ & 1 & 1 & 1 \end{matrix} ; \, 1 \right]_{p-1}$ for prime $p > 3$? Write the pattern and formulate some ideas for proving it. What kinds of implication on the Galois side?

References

[1] Michael Allen, Brian Grove, Ling Long, Esme Rosen, and Fang-Ting Tu. The EHMM Calculator. `https://sites.google.com/view/esme-rosen/ehmm-calculator`, 2026. [Online; accessed 24 July 2026].

[2] Michael Allen, Brian Grove, Ling Long, and Fang-Ting Tu. The Explicit Hypergeometric-Modularity Method I. *Adv. Math.*, 487, Paper No. 110411, 2025.

[3] Victor V. Batyrev. Variations of the mixed Hodge structure of affine hypersurfaces in algebraic tori. *Duke Math. J.*, 69(2):349–409, 1993.